

NASA TECHNICAL
MEMORANDUM

NASA TM X-53100

JULY 28, 1964

NASA TM X-53100

N64-30192
(ACCESSION NUMBER)
10
(PAGES)
TMX-53100
(NASA CR OR TMX OR AD NUMBER)

(THRU)
1
(CODE)
22
(CATEGORY)

**SPACE VEHICLE GUIDANCE -
A BOUNDARY VALUE
FORMULATION**

**PART II: BOUNDARY
CONDITIONS WITH
PARAMETERS**

by ROBERT SILBER
Aero-Astroynamics Laboratory
NASA

*George C. Marshall
Space Flight Center,
Huntsville, Alabama*

OTS PRICE

XEROX \$ 1.10 ph
MICROFILM \$

TECHNICAL MEMORANDUM X-53100

SPACE VEHICLE GUIDANCE — A BOUNDARY VALUE FORMULATION

PART II: BOUNDARY CONDITIONS WITH PARAMETERS

By

Robert Silber

George C. Marshall Space Flight Center

Huntsville, Alabama

ABSTRACT

This report contains an extension of the results presented in NASA Technical Memorandum X-53059, "Space Vehicle Guidance - A Boundary Value Formulation," by Robert W. Hunt and Robert Silber, June 8, 1964. In that memorandum, the control laws for space vehicle guidance were formulated as a set of functions implicitly defined by a set of boundary conditions. In this report the domain of the control laws is augmented to contain mission parameters. In this way, the control laws are defined for a family of missions rather than for a single mission.

20192

Author

NASA-GEORGE C. MARSHALL SPACE FLIGHT CENTER

TECHNICAL MEMORANDUM X-53100

SPACE VEHICLE GUIDANCE — A BOUNDARY VALUE FORMULATION

PART II: BOUNDARY CONDITIONS WITH PARAMETERS

By

Robert Silber

AERO-ASTRODYNAMICS LABORATORY

TECHNICAL MEMORANDUM X-53100

SPACE VEHICLE GUIDANCE - A BOUNDARY VALUE FORMULATION PART II: BOUNDARY CONDITIONS WITH PARAMETERS

INTRODUCTION AND SUMMARY

This report contains an extension of the results of reference one. In reference one, the control laws for space vehicle guidance are formulated as a set of functions implicitly defined by a set of boundary conditions imposed on the solutions of the differential equations of motion and of optimal control. The boundary conditions themselves stem from two sources, the mission criteria and certain conditions necessary for optimality. The implicitly defined control laws have for their arguments the current state of the vehicle, vehicle performance parameters and, possibly, time. A numerical method for the generation of truncated Taylor's series for the control laws is presented in reference one.

The extension of the results of reference one consists of augmenting the domain of the control laws to contain certain "mission parameters." In this way, the control laws are defined for a family of missions rather than for a single mission. Each mission is assumed to differ from the others of the family by the alteration of one or more mission parameters.

As an example, one might consider the mission of injecting into an orbit with given orbital elements. Using the extended domain, the desired orbital elements can be made to appear parametrically in the control laws (and thus also in their Taylor's series) permitting a change of mission either on the pad or in flight.

It is assumed that the reader is familiar with reference one, and throughout this report reference is made to particular sections and equations appearing therein.

BOUNDARY CONDITIONS WITH PARAMETERS

In reference one, the boundary conditions are given by equations (1.22) and later discussed in equation (2.1).

We repeat them here.

$$F_j(t, y_1, y_2, \dots, y_n) = 0, \quad j = 1, 2, \dots, k+1. \quad (1.1)$$

We now replace these functions with the set (1.2)

$$G_j(t, y_1, y_2, \dots, y_n, c_1, c_2, \dots, c_p), \quad j = 1, 2, \dots, k+1.$$

In part two of reference one, the set V is defined as the collection of real $n+1$ -tuples at which each of the functions $F_j(t, y_1, y_2, \dots, y_n)$ is analytic. Correspondingly, we now define W to be the set of all real $n+1+p$ -tuples at which each of the functions $G_j(t, y_1, y_2, \dots, y_n, c_1, c_2, \dots, c_p)$ is analytic.

The definition of the proper, real, analytic, non-singular, controllable solution is altered to read

(i) $\varphi_1(t)$ is real valued on $[\tau^*, t_f^*]$ for each $i=1, 2, \dots, n$ and satisfies equation (2.2) of reference one there;

(ii) For each $t \in [\tau^*, t_f^*]$, the point $(\varphi_1(t), \varphi_2(t), \dots, \varphi_n(t), t) \in U$, and there exists a set of parameters, $(c_1^*, c_2^*, \dots, c_p^*)$, such that $(t_f^*, \varphi_1(t_f^*), \varphi_2(t_f^*), \dots, \varphi_n(t_f^*), c_1^*, c_2^*, \dots, c_p^*) \in W$;

(iii) $G_j(t_f^*, \varphi_1(t_f^*), \varphi_2(t_f^*), \dots, \varphi_n(t_f^*), c_1^*, c_2^*, \dots, c_p^*) = 0$ for each $j = 1, 2, \dots, k+1$;

(iv) $G_j(t, \varphi_1(t), \varphi_2(t), \dots, \varphi_n(t), c_1, c_2, \dots, c_p) \neq 0$ simultaneously for all j for $t \in [\tau^*, t_f^*)$;

(v) A certain Jacobian $J \neq 0$.

The Jacobian J is essentially the same determinant. One forms the composite functions $G_j(t, Y_1(t, \tau, \eta_1, \eta_2, \dots, \eta_{m+k}), \dots, Y_n(t, \tau, \eta_1, \eta_2, \dots, \eta_{m+k}), c_1, c_2, \dots, c_p)$ for each $j = 1, 2, \dots, k+1$ and denotes these functions by $G_j^*(t, \tau, \eta_1, \dots, \eta_m, \eta_{m+1}, \dots, \eta_{m+k}, c_1, c_2, \dots, c_p)$.

The Jacobian J is then given by

$$J = \frac{\partial(G_1^*, G_2^*, \dots, G_{k+1}^*)}{\partial(t, \eta_{m+1}, \dots, \eta_{m+k})},$$

evaluated for arguments corresponding to $t=t_f^*$, $\tau=\tau^*$, $\eta_i=\eta_i^*$ for each $i=1,2,\dots,n$ and $c_i=c_i^*$ for each $i=1,2,\dots,p$.

One now considers the system of equations

$$G_j^*(t, \tau, \eta_1, \dots, \eta_m, \eta_{m+1}, \dots, \eta_{m+k}, c_1, c_2, \dots, c_p) = 0$$

for each $j=1,2,\dots,k+1$. These equations implicitly define the functions

$$\left. \begin{aligned} \eta_{m+r} &= \beta_r(\tau, \eta_1, \dots, \eta_m, c_1, \dots, c_p), \quad r=1,2,\dots,k \\ \text{and} \quad t &= t_f(\tau, \eta_1, \dots, \eta_m, c_1, \dots, c_p) \end{aligned} \right\} \quad (1.3)$$

such that the functions are uniquely defined and analytic in all their arguments in a complex neighborhood of $(\tau^*, \eta_1^*, \dots, \eta_m^*, c_1^*, \dots, c_p^*)$, such that

$$\beta_r(\tau^*, \eta_1^*, \dots, \eta_m^*, c_1^*, \dots, c_p^*) = \eta_{m+r}^*, \quad r=1,2,\dots,k,$$

$$t_f(\tau^*, \eta_1^*, \dots, \eta_m^*, c_1^*, \dots, c_p^*) = t_f^*,$$

and such that

$$\begin{aligned} G_j^* \left[t_f(\tau, \eta_1, \dots, \eta_m, c_1, \dots, c_p), \tau, \eta_1, \dots, \eta_m, \right. \\ \left. \beta_{m+1}(\tau, \eta_1, \dots, \eta_m, c_1, \dots, c_p), \dots, \right. \\ \left. \beta_{m+k}(\tau, \eta_1, \dots, \eta_m, c_1, \dots, c_p) \right] = 0 \end{aligned} \quad (1.4)$$

for each $j=1,2,\dots,k+1$ and all arguments in a complex neighborhood of $(\tau^*, \eta_1^*, \dots, \eta_m^*, c_1^*, \dots, c_p^*)$.

Equations (1.3) furnish the control laws with augmented domain. To reflect this augmentation in the Taylor's series, it is necessary to determine, in addition to the partials treated in reference one, the partials of β_r and t_f with respect to c_i for each $r=1,2,\dots,k$ and for each $i=1,2,\dots,p$.

The partials of the control laws with respect to the arguments $\tau, \eta_1, \eta_2, \dots, \eta_m$ are found exactly as in reference one. It is clear that the presence of the additional parameters will in no way alter the methods of reference one, because of the simple relation

$$F_j(t, y_1, y_2, \dots, y_n) = \quad (1.5)$$

$$G_j(t, y_1, y_2, \dots, y_n, c_1^*, c_2^*, \dots, c_p^*)$$

for each $j=1,2,\dots,k+1$. The F_j are the mission criteria of reference one, and since the partials are all to be evaluated at the end point of the reference, the functions $F_j(t, y_1, y_2, \dots, y_n)$ and $G_j(t, y_1, y_2, \dots, y_n, c_1^*, c_2^*, \dots, c_p^*)$ are completely interchangeable as regards the first $n+1$ arguments.

Once the computation described in reference one has been carried out for the determination of the partials of a given order of the control laws with respect to the arguments $t_f, \eta_1, \dots, \eta_m$, very little additional computation is needed to generate numerical values for the partials of the control laws with respect to the mission parameters (i.e., the c_i).

If (1.4) is differentiated with respect to c_1 , there results

$$\frac{\partial G_j^*}{\partial t} \frac{\partial t_f}{\partial c_1} + \sum_{r=1}^k \frac{\partial G_j^*}{\partial \eta_{m+r}} \frac{\partial \beta_{m+r}}{\partial c_1} + \frac{\partial G_j^*}{\partial c_1} = 0 \quad (1.6)$$

and from the definition

$$G_j^*(t, \tau, \eta_1, \dots, \eta_m, \eta_{m+1}, \dots, \eta_{m+k}, c_1, c_2, \dots, c_p) = \quad (1.7)$$

$$G_j(t, Y_1(t, \tau, \eta_1, \dots, \eta_{m+k}), \dots, Y_n(t, \tau, \eta_1, \dots, \eta_{m+k}), c_1, c_2, \dots, c_p),$$

we have

$$\frac{\partial G_j^*}{\partial t} = \frac{\partial G_j}{\partial t} + \sum_{s=1}^n \frac{\partial G_j}{\partial y_s} \frac{\partial Y_s}{\partial t} , \quad (1.8)$$

$$\frac{\partial G_j^*}{\partial \eta_{m+r}} = \sum_{s=1}^n \frac{\partial G_j}{\partial y_s} \frac{\partial Y_s}{\partial \eta_{m+r}} , \quad (1.9)$$

$$\frac{\partial G_j^*}{\partial c_1} = \frac{\partial G_j}{\partial c_1} . \quad (1.10)$$

Combining (1.6), (1.8), (1.9), and (1.10),

$$\left[\frac{\partial G_j}{\partial t} + \sum_{s=1}^n \frac{\partial G_j}{\partial y_s} \frac{\partial Y_s}{\partial t} \right] \frac{\partial t_f}{\partial c_i} \quad (1.11)$$

$$+ \sum_{r=1}^k \sum_{s=1}^n \frac{\partial G_j}{\partial y_s} \frac{\partial Y_s}{\partial \eta_{m+r}} \frac{\partial \beta_{m+r}}{\partial c_i} + \frac{\partial G_j}{\partial c_i} = 0 .$$

Let i be fixed between 1 and p and let (1.11) be written for each $j=1,2,\dots,k+1$. Then the result is a linear system of $k+1$ equations in the $k+1$ unknown partials

$$\frac{\partial t_f}{\partial c_i}, \frac{\partial \beta_{m+r}}{\partial c_i}, \quad r = 1, 2, \dots, k .$$

In view of the methods of reference one, every other quantity in the linear system can be assumed known. Further, the Jacobian of the system is exactly the determinant J assumed different from zero at the end point of the reference trajectory. Thus, a numerical determination of the partials of the control laws with respect to the mission parameters can be effected for first order partials. Subsequent differentiations of (1.11) will yield linear systems which define higher order partials, as well.

REFERENCE

1. NASA TM X-53059, "Space Vehicle Guidance - A Boundary Value Formulation," by Robert W. Hunt and Robert Silber, June 8, 1964.

APPROVAL

TM X-53100

SPACE VEHICLE GUIDANCE - A BOUNDARY VALUE FORMULATION
PART II: BOUNDARY CONDITIONS WITH PARAMETERS

By

Robert Silber

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This report has also been reviewed and approved for technical accuracy.



R. F. HOELKER, Chief
Astrodynamics & Guidance Theory Division



E. D. GEISSLER, Director
Aero-Astrodynamics Laboratory

DISTRIBUTION

DIR, Mr. F. Williams	R-AERO, Mr. Hart
R-FP, Dr. Ruppe	Mr. Lovingood
I-I/IB-DIR, Col. James	Mr. Winch
	Mr. Tucker
	Mr. Hooper
R-P&VE, Mr. Swanson	Mrs. Chandler
Mr. Goerner	Mr. Lisle
Dr. Krause	Mr. Kurtz
Mr. Burns	Mr. Fine
	Mr. Stone
	Mr. deFries
R-ASTR, Mr. Moore	Mr. Teague
Mr. Richard	Mr. Cummings
Mr. Gassaway	Dr. Sperling
Mr. Taylor	Dr. Heybey
Mr. Brooks	Mr. Jean
Mr. Hosenthien	Mr. Felker
Mr. Scofield	
Mr. Woods	R-RP-U, Mr. Bland
Mr. Digesu	
Mr. R. Hill	AST-S, Dr. Lange
Mrs. Neighbors	
Dr. R. Decher	R-DIR, Mr. Weidner
	Dr. McCall
R-COMP, Dr. Arenstorff	
Mr. Davidson	MS-IP, Mr. Ziak
Mr. Harton	
Mr. Schollard	MS-IPL, (8)
Mr. Seely	
Mr. Calhoun	MS-H
R-AERO, Dr. Geissler	CC-P
Dr. Hoelker (15)	
Dr. Speer	HME-P
Mr. Miner (25)	
Mr. Braud	
Mr. Schmieder	
Mr. Dearman	
Mr. Schwaniger	
Mr. Ingram	
Mr. G. Herring	
Mr. Powers	
Mr. Orser	
Mr. Callaway	
Mr. Thomae	
Mr. Baker	
Mr. Stolze	
Mr. W. Vaughan	